Similarity and Approximate Solutions of Laminar Film Condensation on a Flat Plate

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Laminar film condensation of a saturated pure vapor in forced flow over a flat plate is analyzed as boundary layer solutions. Similarity solutions for some real fluids are presented as a function of modified Jakob number ($C_{pl}\Delta T/Prh_{fg}$) with property ratio ($\sqrt{\sigma_l\mu_l/\rho_v\mu_v}$) and Pr as parameters and compared with approximate solutions which were obtained from energy and momentum equations without convection and inertia terms in liquid flow. Approximate solutions agree well with the similarity solutions when the values of modified Jakob number are less then 0.1 near 1 atmospheric pressure.

Key Words: Laminar Film Condensation, Approximate Solutions, Similarity Solutions

Nomenclature

C_f	: Friction factor
C_{pl}	: Specific heat of liquid [kJ/kg℃]
f, F	: Dimensionless stream function
h	: Local heat transfer coefficient [W/m^{2}K]
h_{fg}	: Latent heat [kJ/kg]
Ja/Pr	: Modified Jakob number, $C_{pl}\Delta T/Prh_{fg}$
k	: Thermal conductivity [kW/m℃]
$\dot{m}$	: Condensation flow rate [kg/m^{2}s]
Pr	: Prandtl number, ν/α
R	: Property ratio, $\sqrt{\rho_l\mu_l/\rho_v\mu_v}$
Re	: Reynolds number, $U_\infty x/\nu$
Nu	: Local Nusselt number, hx/k
T	: Temperature [℃]
U	: Velocity component in x-direction [m/s]
V	: Velocity component in y-direction [m/s]

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(Manuscript **Received** November 27, 2000; **Revised** June 4, 2001)

Greek Symbols

δ	: Liquid film thickness
η	: Similarity variable
θ	: Dimensionless temperature, $(T-T_s)/(T_w-T_s)$
μ	: Viscosity [kg/ms]
ν	: Kinematic viscosity [m^2/s]
ρ	: Density [kg/m^3]
τ	: Shear stress [N/m^2]
φ	: Stream function

Subscripts

f	: Freezing point
l	: Liquid
v	: Vapor
i	: Liquid vapor interface
max	: Maximum
s	: Saturation
w	: Wall
δ	: At the liquid-vapor interface
∞	: Free stream

1. Introduction

Recent engineering developments in aerospace planes, nuclear reactors, etc., require knowledge of fluid mechanics and heat transfer in the condensation processes under forced flow. Laminar film condensation of saturated pure vapor in forced flow on a flat plate is analyzed as boundary-layer flows in liquid and vapor phases.

Carpenter and Colburn(1951) suggested that the major force acting on the condensate film is the interfacial shear at the liquid-vapor interface and that the shear stress on the condensate film is affected by the momentum change of the condensing vapor. However, it is not easy to know the interfacial shear, because it is coupled with the interfacial mass transfer due to condensation. Cess(1960) presented uniform property boundary layer solutions, obtained by means of similarity transformations by neglecting the inertia and energy convection effects within the condensate film and assuming that interfacial velocity is negligible in comparison with the free stream vapor velocity. Jacobs(1966) used an integral method to solve the boundary layer flows by matching the mass flux, shear stress, temperature and velocity at the interface. The inertia and convection terms in boundary layer equations of the liquid film were neglected. Koh(1962) and Lee and Yuen(1987) treated the problem of as an exact boundary layer solution. Lee(1986) has reported the approximate integral solutions on this subject over a flat plate and at entrance region with assumption of a variable liquid viscosity.

The object of this investigation is to find the ranges over which the simple approximate solutions may be used satisfactory instead of the similarity solutions. Generally laminar condensate film is so thin that the inertia and thermal convection terms in liquid flow may be neglected and that simplified approximate solution may replace the similarity solutions(Lee and Lee 1992).

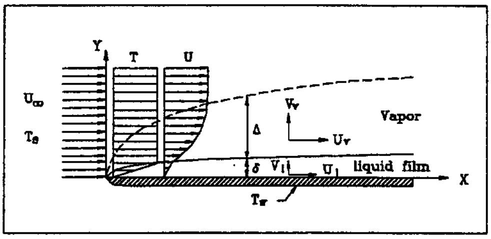

Fig. 1 Physical model and coordinates

2. Governing Equations

Figure 1 shows a sketch of the physical model and coordinate system used for the present study. A mainstream of vapor at a velocity U_∞ is flowing parallel to the wall direction(x) and the velocity distribution is uniform. The vapor is at saturation temperature T_s. The wall surface temperature T_w is constant and lower than T_s and hence condensation takes place. It is assumed that, in steady state, there exists a wave-free laminar liquid film adjacent to the wall surface. Boundary layer flows are assumed in the immediate neighborhood of the interfaces of liquid-vapor and liquid-solid, and potential flow is assumed in the outside region of the vapor boundary layer. The normal direction(y) momentum jump due to mass conversation at the liquid-vapor interface is assumed to be negligible compared with other terms. For laminar two dimensional steady flow, buoyancy and energy dissipation effects are neglected.

By use of the Blasius-type similarity transformation(Koh, 1962), the partial differential governing equations and their boundary conditions can be transformed into the following set of ordinary differential equations.

Similarity variables and stream function :

$$\eta_l = y\sqrt{U_\infty/x\nu_l},\ \eta_v = (y-\delta)\sqrt{U_\infty/x\nu_v} \quad (1)$$
$$\varphi_l = \sqrt{U_\infty \nu_l x}\, f(\eta_l),\ \varphi_v = \sqrt{U_\infty \nu_v x}\, f(\eta_v)$$

Liquid film :

$$f''' + 1/2 ff'' = 0,\ f(0) = f''(0) = 0 \quad (2a)$$
$$\theta'' + 1/2 Pr\theta' = 0,\ \theta(0) = 1,\ \theta(\eta_\delta) = 0 \quad (3a)$$

Vapor layer :

$$F''' + 1/2 FF'' = 0,\ F'(\infty) = 1 \quad (4)$$

Boundary condition at the liquid-vapor

interface :

$$F(0)=Rf(\eta_s),\ F'(0)=f'(\eta_s),\ F''(0)=Rf''(\eta_s) \tag{5}$$

$$\frac{C_{pl}(T_s-T_w)}{h_{fg}Pr}=-\frac{f(\eta_s)}{2\theta'(\eta_s)} \tag{6}$$

Where $f(\eta_l)$ and $F(\eta_v)$ are dimensionless stream functions in liquid and vapor flows and $\theta(\eta_l)$ is dimensionless temperature and prime (′) means differentiation with respect to similarity variables. The similarity transformation is introduced and similarity solutions(Koh, Lee and Yuen) can be obtained as a function of three parameters of Pr, $\sqrt{\rho_l\mu_l/\rho_v\mu_v}$ and $C_{pl}\Delta T/Prh_{fg}$, or corresponding dimensionless thickness of condensate η_s, instead of $C_{pl}\Delta T/Prh_{fg}$,, may be used together with Pr and $\sqrt{\rho_l\mu_l/\rho_v\mu_v}$(Koh, 1962).

3. Simplified Equations

Usually laminar condensate film is so thin that the inertia and thermal convection terms in liquid flow may be neglected. Under these simplifications, approximate solutions can be easily calculated and then compared with similarity solutions. The simplified ordinary differential equations from the above similarity equations can be written as follows (Lee and Lee 1992).

$$f'''(\eta_l)=0 \tag{2b}$$

$$\theta''(\eta_l)=0 \tag{3b}$$

$$F'''+1/2FF''=0 \tag{4}$$

Boundary conditions are same as the case of the similarity governing equations. Methods of solutions are very simple in this simplified case. Simplified approximate solutions, i. e., solutions of non-dimensional liquid velocity distribution, $f'(\eta_l)$ and temperature profiles, $\theta(\eta_l)$ are linear.

The thermo-physical properties are evaluated in each computation at a reference film temperature approximately defined as $T_{film}=T_w+1/3(T_s-T_w)$ at 1 atmospheric pressure. The parameters are shown on Table 1.

4. Methods of Solutions

It is helpful to point out that the momentum Eqs. (2a) and (4) are independent of the energy Eq.n (3a) of liquid film. Hence, the values of three parameters (Pr, $\sqrt{\rho_l\mu_l/\rho_v\mu_v}$ and $C_{pl}\Delta T/Prh_{fg}$) can be computed from any given values of wall temperature (T_w) and property of fluid. Modified Jakob number is implicitly related to the dimensionless film thickness η_s by Eq. (6). First assuming value of η_s, the above momentum Eqs. (2a) and (4) can be solved numerically as a boundary value problem by guessing $f'(\eta_s)$ and η_s and repeating it until solutions are satisfactory to all of the boundary conditions. Once this is done, the energy Eq. (3a) can be readily computed. The solution method of boundary value problems can be used to solve the momentum Eqs. of (2a) and (4) with their boundary conditions of Eq. (5) for any given η_s and $R(=\sqrt{\rho_l\mu_l/\rho_v\mu_v})$. The guessed values of η_s may be found its correct value to satisfy the energy equation and all boundary conditions including equation(6) by trial and error method.

Once the boundary-layer equations are solved, the values of $f'(\eta_s)$ $f''(0)$, $f(\eta_s)$ and $\theta(0)$ are also available. The interfacial velocity, condensate flow rate, skin friction and heat transfer can then be computed by the following equation.

Interfacial velocity It is useful to note that, in terms of the transformed variables, velocity component U are expressible as

$$U=\partial\varphi_l/\partial\eta=U_\infty f'$$

$$U_i/U_\infty=f'(\eta_s) \tag{7}$$

Dimensionless interfacial velocity U_i/U_∞ is obtained from the numerical solutions of $f'(\eta_s)$ and is dependent on the dimensionless liquid film thickness η_s(i. e., dependent on temperature difference T_s-T_w because η_s is determined by $C_{pl}(T_s-T_w)/(h_{fg}Pr)$).

Dimensionless condensate flow rate

$$\frac{\dot{m}}{\rho_l U_\infty}\sqrt{Re_x}=\frac{f(\eta_s)}{2} \tag{8}$$

where $\dot{m}$ is the mass condensed per unit area and unit time.

Interfacial skin friction

$$\frac{1}{2}C_f\sqrt{Re_x}=f''(\eta_s) \tag{9}$$

Table 1 Estimation of Parameters Pr, R, Ja/Pr and Limitation at 1 atm

Fluids \ Parameters	T_s [℃]	T_f [℃]	Pr	R	$(Ja/Pr)_{max}$	$(\eta_\delta)_{max}$	Remarks
Water	100.0	0.0	2.45	231	0.038	2.01	Group 1 (Ja/Pr low values)
R-113	47.56	-34.95	9.20	351	0.041	2.022	
Ethanol	78.3	-114.5	16.3	266	0.016	1.963	
Glycerin	290.0	18.0	423.5	806	0.012	1.60	
Ethylene -Glycol	197.0	-12.3	19.0	213	0.021	1.974	
Mercury	356.95	-38.83	0.012	233	9.78	5.266	Group 2 (Liquid metal)

where

$$C_f = \frac{\mu_l(\partial U_l/\partial y)_\delta}{1/2\rho_l U_\infty^2} \tag{10}$$

Heat Transfer The local heat transfer per unit time and per unit area is given by $q''=k_l(\partial T/\partial y)_{y=0}$ in which heat flux $q''(=\dot{m}h_{fg})$ is positive into surface. The above heat flux expression becomes

$$q''=k_l(T_s-T_w)\sqrt{U_\infty/\nu_l x}\,\theta'(0) \tag{11}$$

$$\frac{Nu_x}{\sqrt{Re_x}}=-\theta'(0) \tag{12}$$

where

$$Nu_x=\frac{hx}{k} \tag{13}$$

$$h=\frac{q''}{T_s-T_w} \tag{14}$$

5. Numerical Results

Once the boundary layer equations are solved, the numerical values for dimensionless stream functions of liquid and vapor flow, velocity and temperature profiles of condensate are available. The dimensionless liquid-film thickness η_δ is implicitly related to the dimensionless physical group $C_{pl}\Delta T/Prh_{fg}$ by Eq. (6).

5.1 Dimensionless thickness of condensate (η_δ) vs. modified Jakob number

The present problems involve three physical parameters of Pr, $R(=\sqrt{\rho_l\mu_l/\rho_v\mu_v})$ and $C_{pl}\Delta T/Prh_{fg}$. Figure 2 shows the relation between modified Jakob number $C_{pl}\Delta T/Prh_{fg}$ (also defined as Ja/Pr) and $\eta_\delta(=\delta\sqrt{U_\infty/x\nu_l})$ and limitation of the range of solutions at 1 atmospheric pressure (see Table 1).

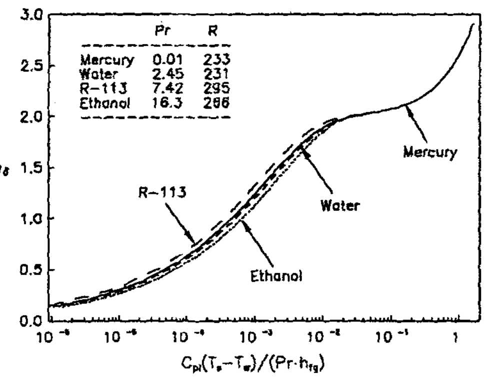

Fig. 2 Comparison of modified Jakob number Ja/Pr and liquid film thickness η_δ

Two groups are suggested depending on the high and low ranges of the values of η_δ or corresponding values of $C_{pl}\Delta T/Prh_{fg}$ computed from Eq. (6). Low value cases ($\eta_\delta<2.1$ equivalent to $Ja/Pr<0.1$) will be called Group 1. High value cases ($\eta_\delta\geq 2.1$ equivalent to $Ja/Pr\geq 0.1$) will belong to Group 2 (liquid metals). Most of the condensing fluids belong to Group 1 except liquid metal cases(possibly liquid metals belong to both Group 1 for small ΔT and also Group 2 for large ΔT cases) at 1 atm.

5.2 Velocity profiles

Figure 3(a) shows that the solutions of liquid velocity profiles for water are basically linear and directly depend on liquid-film thickness η_δ. For

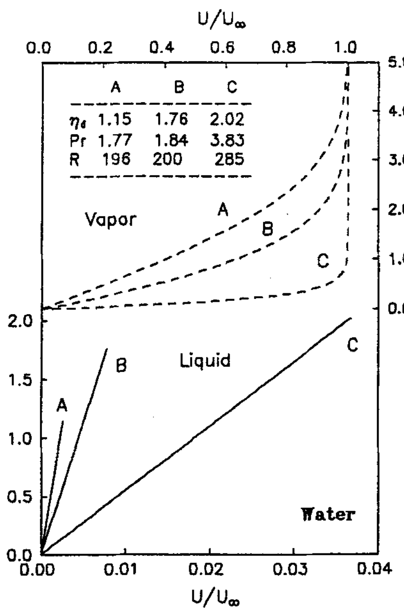

Fig. 3 (a) Detail velocity profile of liquid water and water vapor

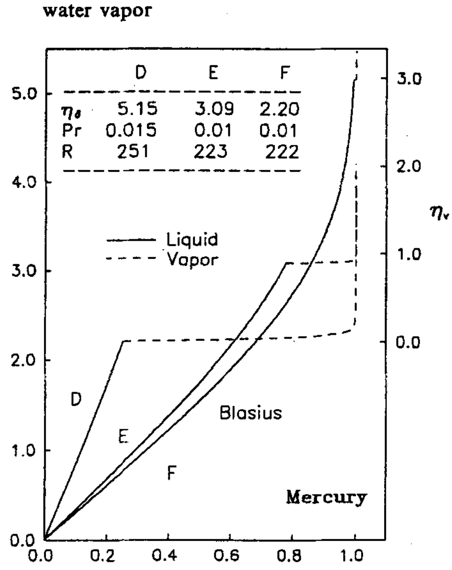

Fig. 3 (b) Velocity profiles for mercury

mercury, the liquid velocity profiles, as shown in Fig. 3(b), are linear when the values of η_δ are less then 2.1 and also very sensitive to the variations of the wall temperature. If the values of η_δ are larger than 2.2, the liquid film becomes relatively thick and the liquid velocity profiles become more nonlinear (Group 2 case).

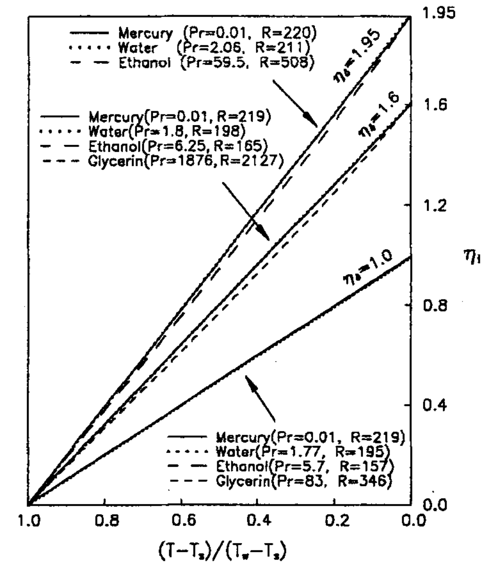

Fig. 4 Comparison of temperature profiles of liquid condensate

5.3 Temperature profiles

For the selected fluids, the temperature profiles corresponding to the values of liquid-film thickness(η_δ) are presented in Fig. 4 at 1 atmospheric pressure. Non-linearity of the temperature profile through the condensate film depends on the values of Prandtl number(Pr) and η_δ. Cases for large Pr and large η_δ show more nonlinearity. However, the deviation between similarity and approximate solutions is found to be within 3% at 1 atm.

5.4 Interfacial velocity, condensate flow rate, skin friction and heat transfer

Figures 5, 6, 7 and 8 show the comparison between the approximate and similarity solutions. Dimensionless values of interfacial velocity, condensation flow rate, local skin friction and heat transfer coefficients for the water(belong to group 1) and mercury(belong to group 2) can be read directly as a function of the liquid film thickness η_δ or Ja/Pr from Figs. 5, 6. 7 and 8 respectively. From Figs. 5, 6 and 7, it is found that the interfacial velocity, condensation flow rate and skin friction increase as the liquid film thickness increases. For the water with liquid film thickness $\eta_{\delta\max}<2.1$, the approximate solutions are nearly

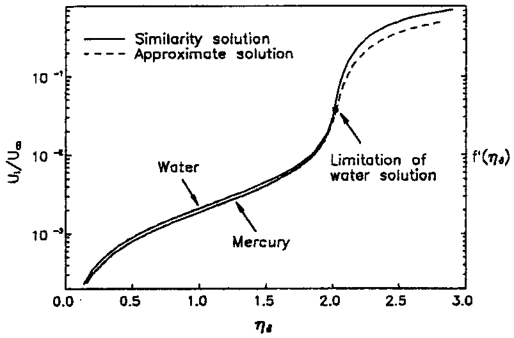

Fig. 5 Liquid film thickness η_δ and interfacial velocity

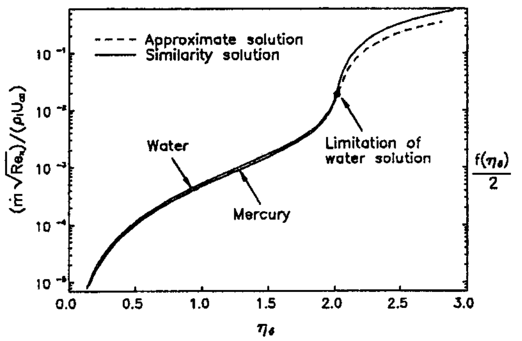

Fig. 6 Liquid film thickness η_δ and dimensionless condensate flow rate

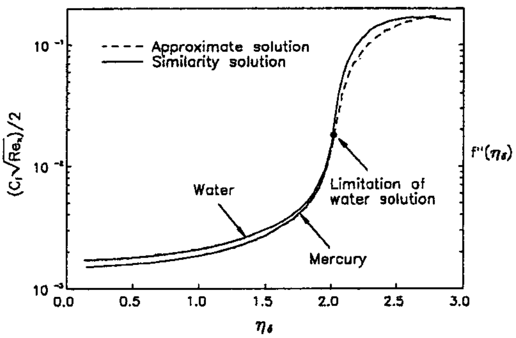

Fig. 7 Liquid film thickness η_δ and interfacial skin friction

the same as the similarity solutions. But for the mercury case, the deviation between similarity and approximate solutions increases when the values of η_δ is greater than 2.1.

Figure 8 shows that the dimensionless heat transfer coefficients for water and mercury decreases as the values of $C_{pl}\Delta T/Prh_{fg}$ increases.

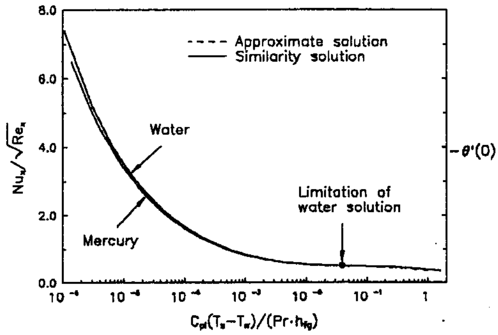

Fig. 8 Modified Jakob number Ja/Pr and local Nusselt number

Fig. 8 also shows that the Nusselt number decreases monotonically as the liquid film thickness increases. Approximate solutions of the heat transfer for both of water and mercury are well agreed with the similarity solutions.

6. Conclusions

The two-phase boundary-layer equations in laminar film condensation for flow over a flat plate for some real fluids have been solved numerically and compared with approximate solutions. It was found that the energy transfer by convection and the effects of inertia term in the momentum equation of the liquid flow are negligibly small at 1 atmospheric pressure and that hence the approximate solutions may be satisfactory applied to the most of the fluids except liquid metals. For liquid metals, the values of $C_{pl}\Delta T/Prh_{fg} \geq 0.1$(equivalent to $\eta_\delta \geq 2.1$), and exact similarity solutions are recommended.

Acknowledgment

This work was partially supported by the Science Research Fund of School of Mechanical Engineering, Pusan National University.

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